As-Fab Elements – Design vs. Reality

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ABSTRACT

In the design of optical assemblies, emphasis is placed on tolerancing the surface irregularity, which is a driving factor in price and manufacturing prices and time during polishing. Quite often, the default irregularity tolerance in modeling software is assumed to be a 50:50 split between astigmatism and 3rd order spherical aberration (i.e. symmetric zonal errors). In this paper, we reviewed the irregularity of over 1,000 custom fabrication optical surfaces. We looked at the relationship between the spherical and astigmatism aberrations and found generally that a surface will be either astigmatic or spherical, but rarely a mixture of the two. We also looked at the PV and rms of the surfaces and how that compares to the model and the general knowledge. One striking result of our analysis came from a closer analysis of how the optical modeling software package handles ‘power’ errors in the irregularity tolerance. It is possible that there is a mismatch between the model and the optical manufacturer.

Keywords: Tolerancing, Surface Irregularity, Optical Fabrication

1. INTRODUCTION

The use of raytracing software is commonplace in optical engineering and allows the generation and testing of mathematical models by the optical designer. After the optical design is shown to meet necessary performance criterion, a tolerance and sensitivity analysis is often done to consider fabrication and assembly errors which typically cause optical performance degradation. Common assembly errors in optical systems include lens element positional errors and common fabrication errors which include errors in the lens radius of curvature, center thickness, and surface irregularities such as astigmatism and spherical.

The robustness and manufacturability of the optical design is often qualitatively indicated by the results of the sensitivity analysis but can be further quantitatively modeled by Monte-Carlo analysis. In this analysis, multiple systems are analyzed each with a different set of feasible assembly and fabrication errors (pulled from a distribution). The outcome is then an optical system performance over a range of possible errors. The overall accuracy of this Monte Carlo analysis is therefore influenced by the accuracy of the modeled errors.

We are focusing on the irregularity component of the tolerance analysis to determine how the modeled errors differ from the actual as-fabricated errors and how this may affect our overall system modeling. In a typical tolerance analysis, the irregularity is modeled by assuming an equal combination of spherical and astigmatism. But, upon inspecting the measured irregularities for many different lenses, we find instead a dichotomy – either spherical or astigmatism errors occur.

1.1 Hypothesis

A correlation between the type of lens surface figure errors and manufacturing method and/or lens characteristics is expected. Knowing such a correlation between astigmatism and/or spherical irregularities and lens characteristics a-priori could allow for a more accurate Monte-Carlo analysis. Data collected on the surface irregularities seem to confirm the suspicion of poor correlation between astigmatism and spherical surface irregularities, Figure 1.

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1.2 The Parts

The data presented here are from 52 different lens types made with various optical glasses, curvatures, and diameters; multiple quantities (between 7 to 30) of each lens was made, allowing for multiple samples across the wider breadth of data. A total of 1052 surface irregularity measurements were analyzed. The nominal radii of curvature of the spherical lenses ranged from 6.67 mm to 495.88 mm and the clear aperture ranged from 6 mm to 36 mm. Plano surfaces are also shown. In general, the irregularity tolerance for these lenses was 0.1 waves peak to valley (at 632.8 nm). The optical model analysis showed good performance with a modeled 0.1 wv PV error. Some parts may show higher PV values due to different tolerances. There was no specification on higher order aberration terms.

All lenses in this study were fabricated and tested by Optimax Systems, Inc. [1]. All surfaces were CNC ground to approximate shape and subsequently pitch-polished. Radius of curvature and surface irregularity was tested for each lens on a commercially available phase-shifting Fizeau interferometer equipped with a radius scale (with displacement interferometry). Measurement accuracies are understood to be negligible in the current context. Examples of the irregularity measurements are shown in Figure 2 where (a) shows a surface with significant spherical (and higher order terms) error and (b) shows significant astigmatism.
It is feasible that the different aberrations are due to different fabrication processes and/or setups. For example, surfaces with large R/numbers (radius of curvature/diameter) allow for multiple quantities to be fabricated simultaneously on lapping and polishing machines, known as a “multiple block”. Surfaces with small R/#s can be more difficult to fabricate and generally can only be made as singles – one part per tool. In many cases, parts from the multiple block (large R/#s) will show more astigmatism than spherical (except for the center component). In contrast, parts from the single block will typically show more spherical than astigmatism (small R/#s). We plan to look at the collected data to test this theory.

1.3 Tolerancing Irregularity in Optical Modeling Software

There are several different optical design software packages that handle tolerancing and surface irregularity in different ways. In this work, the surface irregularity was modeled in Zemax and OpticStudio by using the “TIRR” tolerance. OpticStudio’s Help File describes the TIRR irregularity modeling as combination of equal parts spherical and astigmatism [2]. The TIRR command calculates the surface sag error (Δz) for irregularity tolerancing as

\[ Δz = \frac{\lambda w}{4} (\rho^4 + \rho_1^2), \]

where

\[ \rho = \sqrt{(\rho_x^2 + \rho_y^2)} \]

and

\[ \rho_1 = \rho \cos \theta \]

where \( \rho_x \) and \( \rho_y \) are the normalized lateral (radial) coordinate on the height map, \( \theta \) is the angular coordinate, \( \lambda_t \) is test wavelength, and \( W \) is the amount of error assumed in the model in fringes. The angle of the astigmatism (not indicated in the equation) has a random orientation in the Monte Carlo Analysis.

An example of this irregularity surface sag error from the Zemax analysis is shown in Figure 3. In Figure 3(a), the surface sag, Eqn. 1, is plotted directly for an input \( W \) of 0.1 fr (equivalent to 31.6 nm in a standard double pass test in HeNe). The PV of this height map is 32.3 nm. This small difference between the input and the output PV can be explained by the pixelization of the height map and is not of concern. What is of concern is shown in Figure 3(b), where the power error (\( \rho_1^2 \)) is removed. This power error is removed for multiple reasons: because power error is better represented as a radius error, the interferometer test of the irregularity doesn’t measure the power error, and because the surface sag is not normal to the surface as it should be. The output PV of this height map is much more representative of the actual measured PV of a surface with the modeled error. The PV of this is 16.9 nm, about half of the input. Figure 4 shows this same trend for a range of irregularity (W) inputs. In summary, the measured PV will be about half of the modelled PV.

![Figure 3: (a) The surface sag error (Eqn. 1) with an input of 0.1 fr for W and (b) the same surface sag error with the power term removed. Power is sometimes called focus or defocus and here means the \( \rho_1^2 \) term mathematically.](image_url)
Figure 4: The PV output (after removing power errors) for the Zemax modeled irregularity. Note that the output is about half of the input.

The consequence therefore is a mismatch between the model and the as-manufactured part. For example: an optical designer will input 0.1 fr as $W$ in the TIRR analysis. The analysis will be run with 0.05 fr (because of the typical re-focusing in the model) and the designer will determine that the performance meets specification. Then the designer will put the same 0.1 fringes on the optical print for a PV specification thinking that is needed for performance. Then optical manufacturing facility will provide a part with 0.1 fr error, which is double what was actually modeled. The parts may have more error than was actually modelled. This would still hold true for a non-interferometric test because manufacturers typically report irregularity with all power removed.

Perhaps this difference is because optical testing results were previously reported in wavefront, not in surface error. A typical manufacturer now reports irregularity as the surface error, irrespective of the units specified.

This method of tolerancing irregularity (equal spherical and astigmatism) is simplistic, but is the most commonly used method for its ease of use. Zemax and other software packages can do more complex irregularity tolerance analysis where the individual irregularity components (typically Zernike terms) are tolerated separately.

Two points of concern in the default tolerancing are: 1) the use of the Seidel aberrations and 2) the tolerance error is added through a sag deformation instead of along the surface normal (typical during testing). For the later concern, the first effect would be an error in the power (radius of curvature) which would become larger for the smaller R/#s. This power error can be negated at this time. There would be some secondary effects with spherical and high order spherical terms (again, larger with the smaller R/#s). This effect is noted, but is not considered at this time. Further work could include calculating the magnitude of this error, but the better solution would be to use the more complex irregularity tolerancing in the optical modeling software when the error is along the surface normal.

The use of Seidel aberrations is concerning because of their non-orthogonally and their mismatch with the more commonly used Zernike aberrations during optical testing. In the interferometric optical test of spherical and plano surfaces (where power is removed), the Seidels can easily be converted to the Zernike terms as shown in 1.4. Testing results shown below will use the more common Zernike terms which have a relationship with the Seidels as discussed below.

### 1.4 Definition of Terminology

The Zernike aberration definitions used in this paper use the orthogonal, orthonormal set of Zernike polynomials [3] and are:

- **Zernike Spherical Aberration:**
  
  \[ H_{sph} = Z_4^0 \sqrt{5}(6\rho^4 - 6\rho^2 + 1), \]  

- **Zernike Astigmatism X:**
  
  \[ H_{AstigX} = Z_2^2 \sqrt{6}(\rho^2 \cos2\phi), \]
Generally, the absolute value of the Spherical Aberration (SA) is used in the following analysis. We combine the individual Zernike terms in a root-sum-square manner to determine the magnitude of the astigmatism and coma:

Zernike Astigmatism Magnitude:

\[ Z_{2m}^2 = \sqrt{(Z_2^2)^2 + (Z_2^{-2})^2} \]  

(7a)

Zernike Coma Magnitude:

\[ Z_{3m}^1 = \sqrt{(Z_3^1)^2 + (Z_3^{-1})^2} \]  

(7b)

The conversion between the Seidel and Zernike coefficients is a non-trivial problem. In the case that we are considering here – no power or tilt in the measurements and no higher order Zernike terms considered – the following relationships can be made between the Seidel terms (\( S_{xx} \)) and the Zernike terms (\( Z_{xx}^m \)):

Spherical Aberration:

\[ S_{40} = Z_4^0 \frac{6\sqrt{5}}{5} \]  

(8a)

Astigmatism

\[ S_{22} = Z_2^{2m} \frac{2\sqrt{5}}{5} \]  

(8b)

Coma

\[ S_{31} = Z_3^{1m} \frac{6\sqrt{2}}{5} \]  

(8c)

The tolerance relationship in the Zemax TIRR analysis assumes a one-to-one ratio. To compare our measured Zernike terms, an update to this relationship is required as follows:

\[ \frac{S_{40}}{S_{22}} = 1 = \frac{Z_4^0 \frac{6\sqrt{5}}{5}}{Z_2^{2m} \frac{2\sqrt{5}}{5}} = 2.7386 \frac{Z_4^0}{Z_2^{2m}} \]  

(9)

The plotted data shows the Zernike Spherical plotted versus the Zernike Astigmatism. To compare this measured data to the Zemax analysis, we will plot \( Z_4^0 = 0.365 * Z_2^{2m} \) as calculated from equation 9 [2] - [3].

The R/# of a surface is used throughout this paper:

\[ R/# = \frac{\text{Surface Radius of Curvature}}{\text{Surface Aperture Diameter}}. \]  

(10)

2. MEASURED IRREGULARITY RESULTS AND DISCUSSION

The following sections show the data of the as-fabricated lenses in various forms look for trends and compare to the optical modeling software. Most data shows the data sorted by surface type – concave (CC), convex (CX), or plano. All optical surfaces were measured using a commercially available Fizeau interferometer by Optimax. All data has tilt and power removed. Some analysis shows Zernike polynomials (1.4), which were used to examine specific aspects of surface irregularity. The Peak-to-Valley (PV) numbers shown here are full PV in the part’s clear aperture. While PVr [4] or PV99 (masking the lowest and highest data points) are typically better values, these values were not available at this time.

2.1 Spherical vs. Astigmatism

The plots for spherical (absolute value) vs. astigmatism are shown in Figure 5. The Zemax irregularity model (\( Z_4^0 = 0.609 * Z_2^{2m} \)) is shown as the red line. Clearly the data do not follow the trend of the Zemax model. The blue line shows a linear trend of the data. It is clear that the data does not fit well to a linear trend.

There appears to be a tradeoff for concave surfaces between the astigmatism and spherical terms, as suggested by the more triangular grouping of data points. This is the either spherical or astigmatism being the dominating error as we hypothesized. The same story is harder to tell for the convex surfaces. The data does show that plano surfaces are more susceptible to astigmatism than spherical figures errors during the fabrication process, as expected.
Sign of Aberrations

The plots presented here are show the absolute value for spherical aberration because we were initially only looking at the magnitude. When we consider the sign of spherical (when vs. astigmatism or vs. R#) there are no additional conclusions that can be drawn. The sign of the spherical appears to differ when surface shape is considered: 58.7% of the concave surfaces had positive spherical and 49.4% of the convex surfaces had positive. This is interesting, but we do not yet know enough about the manufacturing process to determine if this is due to a process difference or just a statistically anomaly. Also, the difference does not seem to be large enough to force a change in the modeled distribution in the irregularity tolerance model.

The sign of astigmatism, when considering Zernike methods, cannot be separated from the clocking of the aberration. This clocking is not significant to the single surface analysis we are doing here.

2.2 Other aberrations

The standard model only takes spherical and astigmatism into account. A typical part typically has more components of surface error, but these are not easily represented in simple values. Spherical and astigmatism are notable in that traditional manufacturing processes naturally lend themselves to creating those types of errors. Optical errors like coma are more of an assembly error than a manufacturing error (expecting some sub-aperture polishing processes and aspheres which are not considered here). Even so, we plotted the coma terms versus astigmatism to determine if coma might have a more apparent correlation. The data, plotted below in Figure 6, do not seem to suggest any notable relationship. In addition, the magnitude of the coma terms are small compared to the astigmatism and spherical terms.
Figure 6. Coma versus astigmatism. These data do not seem to show strong correlations.

Another way to look at this is to plot the relative amount of aspherical, astigmatism, and coma. This is shown in Figure 7 where the aberration percent for each term is plotted. The aberration percent is:

$$\left( \frac{|\text{Aberration (\mu m)}|}{\text{PV (\mu m)}} \right) \times 100$$

(11)

As shown, spherical is the dominating error and coma is small. For the plano surfaces, the astigmatism appears to be a larger contributor to the PV as compared to the CC and CX surface as expected.
Figure 7. Plots of the percentage of contributions of spherical, coma, and astigmatism coefficients for concave (a), convex (c), and plano (b) surfaces.
2.3 PV and rms

We next looked at the overall PV and rms values to look for trends and compare to other well-known rules of thumb. As shown in Figure 8, many of the surfaces are below the 0.063 µm that corresponds to a λ/10 PV specification. Most of the surfaces in this study were specified with a λ/10 PV. Surfaces that have higher PV had a different PV specifications. As shown, the histograms of the PV values are offset, especially for the CX and CC surfaces. The plano surfaces show a more normal distribution. The CX and CC surfaces show that the distribution is significantly shifted towards the specification value. This is due to the manufacturing process – it is difficult to achieve a zero or close to zero irregularity value and because polishing typically will stop if the PV is within specification (if the other parameters are also within specification). There is no manufacturing advantage to continue to polish to achieve a lower PV value. In fact, it is generally a disadvantage because of the risk of scratching, going center thickness minus, or getting out of radius tolerance.

Figure 8. Histograms of the measured PV.

These histograms can be used to provide feedback to the irregularity modeling. When tolerancing, the optical designer has the option of choosing what distribution to choose. Options will vary between which software package is used, but they generally are normal (Gaussian), uniform, or parabolic. Based on the data in Figure 8, the designer should pick the parabolic distribution when considering that the model can assume a negative PV (which is not physically possible).
PV:rms Ratio

The “known” ratio of PV to rms for an aberration varies greatly depending on who is speaking and which aberration (and which definition) they are speaking of. We have seen ratios ranging from 57:1 to 3:1. Overall though, many people will say when all aberrations in a surface error are considered, historical data will show a near 5:1 ratio. This ratio is for traditionally polished surfaces with ‘older’ interferometers (i.e. lower pixel resolution). The PV:rms ratios of the measured surfaces are shown in Figure 9 and Figure 10 (histogram). As shown, most of our surfaces have higher PV:rms ratios. Mean values are 6.2 for the CC surfaces, 7.5 for the CX surfaces, and 6.7 for Plano surfaces, which is not significantly larger than the rule of thumb. The difference may be explained by improved camera resolutions and noise in the measurement raising the PV, but not the rms as much.

![Figure 9. PV vs rms for all surfaces.](image1.png)

![Figure 10. Histograms of the PV:rms ratio.](image2.png)
2.4 R/# Impact

We suspected that the lens R/# would have an impact on the propensity for a surface to have either spherical or astigmatism due to the ability for larger quantities of large R/# surfaces to be fabricated in parallel while low R/# surfaces require individual tooling to be made. Figure 11 shows the aberrations plotted relative to the surface R/#. As shown, there does not appear to be a strong correlation in this data. The data also has a lack of sampling in certain ranges of R/#’s but the distribution is relatively flat for the parts tested.

![Graph showing aberrations plotted relative to surface R/#. The data shows a lack of correlation and a relatively flat distribution for the parts tested.](image)

Figure 11. This plot shows coefficients measured for astigmatism and spherical in the fabricated lens surfaces with respect to the R/# of the given surface. Plano surfaces are represented by an R/# of 1000.

3. CONCLUSIONS

We analyzed 1052 surface irregularity measurements of as-fabricated optical surfaces to compare to the optical model and to known rules of thumb. One specific lesson learned during this analysis is the need for very close identification of which aberration definition is used (Zernike Fringe, Zernike Standard, vs. Seidel). We have also found what appears to be an inconsistency in the TIRR analysis in Zemax showing that the PV of the modeled irregularity is not the same PV that an optical manufacturer would provide. We also showed that the distribution of the PV errors should probably be modeled by a one-sided parabolic distribution.

The surface figure errors of spherical and astigmatism are not correlated, as some tolerancing and Monte-Carlo methods assume. While concave surfaces may have a negative correlation between the two figure errors it is still not clear what lens design attributes may give rise to one over another as surface R/# does not seem to favor one type of figure error over the other. If it can be known which figure errors or what ratio of figure errors an optic is most likely to exhibit upon
fabrication, those errors can be simulated during the Monte-Carlo analysis to grant a more accurate statistical yield of the as-built systems. In Zemax and OpticStudio, for example, instead of using the TIRR operand to perturb each surface with a random, but equivalent amount of spherical and astigmatic surface departure, the surface could be defined as a Zernike phase surface and the Zernike term and magnitude could be narrowed down to simulate a random amount of only those terms which that particular lens surface is known to be susceptible to during fabrication.

In the future, Ruda-Cardinal, Inc. would like to update previous Monte-Carlo analysis done for the systems using these optics with the as-measured distributions of surface errors and compare the expected performances of the perturbed systems to that of the as-built systems.

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REFERENCES

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